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Uncertainty of complex permittivity measurements by split-post dielectric resonator technique

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Abstract

Split-post dielectric resonators operating at frequencies 1.4–5.5 GHz were used to measure complex permittivity of single crystal standard reference dielectric materials with well known dielectric properties previously measured by other techniques. Detailed error analysis of permittivity and dielectric loss tangent measurements has been performed. It was proved both theoretically and experimentally that using split post resonators it is possible to measure permittivity with uncertainty about 0.3% and dielectric loss tangent with resolution 2×10^{-5} for well-machined laminar specimens. © 2001 Elsevier Science Ltd. All rights reserved.

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1. Split-post dielectric resonator theory

The split-post dielectric resonator (SPDR) described in Refs. 1–7 is already a well-established technique for measurements of the complex permittivity of dielectric and ferrite laminar specimens at frequency range 1–10 GHz. The geometry of split dielectric resonators used in our measurements is shown in Fig. 1.

SPDR typically operate with the $TE_{01\delta}$ mode that has only the azimuthal electric field component so the electric field remains continuous on the dielectric interfaces, what makes the system insensitive to the presence of air gaps perpendicular to *z*-axis of the fixture. Rayleigh–Ritz method was used to compute the resonant frequencies, and the unloaded *Q*-factors and all other related parameters of SPDR's. The real part of permittivity of the sample was found on the basis of measurements of the resonant frequencies and thickness of the sample only as an iterative solution to the Eq. (1).

$$\varepsilon_r' = 1 + \frac{f_0 - f_s}{hf_0 K_\varepsilon(\varepsilon_r', h)} \tag{1}$$

where:

h thickness of the sample under test,
f_0 resonant frequency of empty SPDR,
f_s resonant frequency of the resonator with the dielectric sample
K_ε function of ε_r' and h

K_ε was computed for a number of ε_r' and h for a given resonant fixture and tabulated. Interpolation was used to compute K_ε in the subsequent iterations. The initial guess for K_ε in formulae (1) was taken to be the same as its corresponding value for given sample thickness *h* and $\varepsilon_r' = 1$. Subsequent values of K_ε were found from the subsequent permittivity values obtained in the iterative procedure. Because K_ε is a slowly varying function of ε_r' and *h* so iteration process converges rapidly.

When real permittivity was found then the dielectric loss tangent of the sample was determined from formula (2).

$$\tan\delta = (Q^{-1} - Q_{\mathrm{DR}}^{-1} - Q_c^{-1})/p_{\mathrm{es}} \tag{2}$$

where:

Q unloaded *Q*-factor of the resonant fixture containing dielectric sample
p_{es} electric energy filling factor of the sample defined as

$$p_{\mathrm{es}} = \frac{W_{\mathrm{es}}}{W_{\mathrm{et}}} = \frac{\int\int\int_{V_S} \varepsilon_S \mathbf{E}\cdot\mathbf{E}^* \mathrm{d}v}{\int\int\int_V \varepsilon(v)\mathbf{E}\cdot\mathbf{E}^* \mathrm{d}v} = h\varepsilon_r' K_1(\varepsilon_r', h) \tag{3}$$

Q_c *Q*-factor depending on metal losses for the resonant fixture containing the sample

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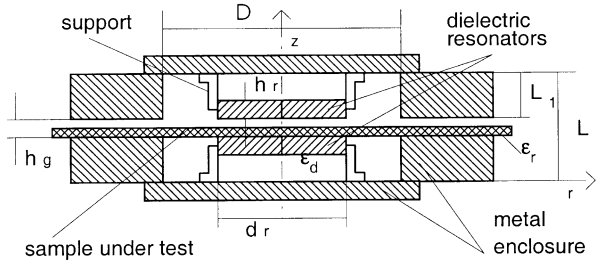

Fig.1. Schematic diagram of a split dielectric resonator fixture.

$$Q_c = \frac{\iiint_V \mu_0 \mathbf{H}\cdot\mathbf{H}^* \mathrm{d}v}{R_S \oiint_S \mathbf{H}_\tau\cdot\mathbf{H}_\tau^* \mathrm{d}s} = Q_{c0} K_2(\varepsilon_r', h) \tag{4}$$

Q_{c0} Q-factor depending on metal losses for empty resonant fixture

$$Q_{\mathrm{DR}} = Q_{\mathrm{DR0}}(f_0/f_s)(p_{\mathrm{eDR0}}/p_{\mathrm{eDR}}) \tag{5}$$

p_{eDR}, p_{eDR0} electric energy filling factors for the dielectric posts of the SPDR, containing a sample and empty respectively,

Q_{DR0} Q-factor depending on dielectric losses in dielectric posts for empty fixture.

Again K_1 and K_2 are functions of ε_r' and h so that they were computed and tabulated. Interpolation was used to compute K_1 and K_2 for current values of h and ε_r'.

2. Theoretical uncertainty analysis

The main source of uncertainty of the real permittivity is related to uncertainty of the thickness of the sample under the test. Relative error of real permittivity due to thickness uncertainty can be expressed as follows:

$$\frac{\Delta\varepsilon_r'}{\varepsilon_r'} = T\frac{\Delta h}{h} \tag{6}$$

where: $1 < T < 2$,

Usually T value is very close to unity except for thick, large permittivity samples. For such samples T value increases but always remains smaller then two. Additional factors affect the overall uncertainty, e.g. differences between real dimensions of the resonant fixture and permittivity of dielectric resonators and the values assumed in computations. All those extra errors can be analyzed indirectly via their influence on the computed values of K_ε according to the equation

$$\frac{\Delta K_\varepsilon}{K_\varepsilon} = T_{dr}\frac{\Delta dr}{dr} + T_{hr}\frac{\Delta hr}{hr} + T_{\varepsilon d}\frac{\Delta\varepsilon d}{\varepsilon d} + T_D\frac{\Delta D}{D} + T_L\frac{\Delta L}{L} + T_{hg}\frac{\Delta \mathrm{hg}}{\mathrm{hg}} \tag{7}$$

Error coefficients T appearing in Eq. (7) can be computed for a specific resonant fixture and properties of the sample. Computed values of error coefficients for our 3.9 GHz fixture are presented in Table 1.

For other fixtures used by us, error coefficients values were similar and they are not presented here. Total uncertainty of K_ε for specific resonant fixture depends on machining precision for particular parts of the resonator and uncertainty of dielectric resonator permittivity. For our 3.9 GHz resonator these uncertainties were as follows:

$$\frac{\Delta dr}{\mathrm{d}r} = 0.1\%,\ \frac{\Delta hr}{hr} = 0.5\%,\ \frac{\Delta\varepsilon d}{\varepsilon d} = 0.2\%,$$
$$\frac{\Delta D}{D} = 0.1\%,\ \frac{\Delta L}{L} = 0.2\%,\ \frac{\Delta hg}{hg} = 0.5\%$$

All estimated uncertainties in this paper are quoted for a coverage factor of $k = 2$, corresponding, approximately, to a 95% confidence level.

Substituting these values into Eq. (7) and assuming that all errors have the same sign one can estimate the upper bound for relative error of K_ε.

It is seen from the data presented above the most significant contribution to the overall K_ε error arise from coefficients T_{hr}, and $T_{\varepsilon d}$ related to the thickness and permittivity of the dielectric resonators. In practice it is possible to mitigate K_ε errors related to those two coefficients by taking into account measured value of the resonant frequency for empty split post resonator. Assuming certain value for thickness of the split post resonators and all other dimensions of the resonant structure it is possible to choose such permittivity of dielectric resonators to get identical computed and measured resonant frequency values for empty fixture. Exact numerical analysis has shown that in such a case K_ε errors due to uncertainty of dielectric resonator thickness and permittivity practically cancel out. If such approach is used it is possible to compute K_ε coefficients for specific resonant structure with uncertainties better then 0.15% so one can estimate the total uncertainty for real permittivity as

$$\frac{\Delta\varepsilon_r'}{\varepsilon_r'} \leqslant 0.15\% + T\frac{\Delta h}{h} \tag{8}$$

Table 1
Computed values of error coefficients for 3.9 GHz split post dielectric resonator

εr	T_{hr}	$T_{\varepsilon d}$	T_{dr}	T_L	T_D	T_{hg}
2	0.853	0.967	0.312	−0.069	−0.239	0.097
10	0.803	0.778	0.432	−0.058	−0.257	0.050

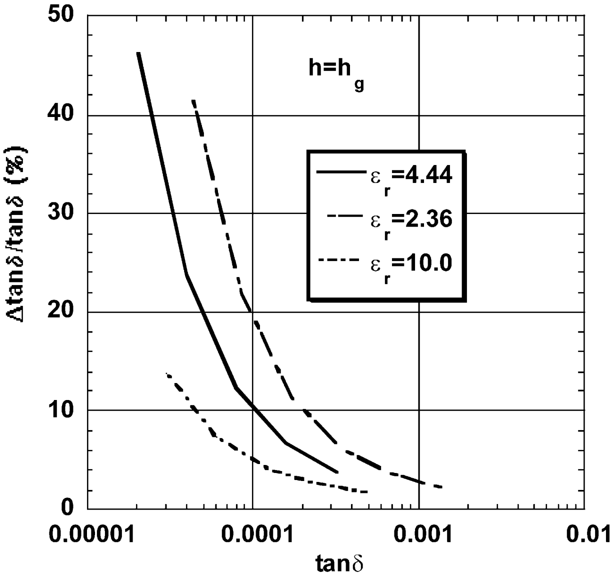

Fig. 2. Uncertainty of dielectric loss tangent measurements using 3.37 GHz SPDR with Q_0-factor 23600 and $hg = 3.00$ mm assuming Q-factor measurements uncertainty 1%.

In principle it is possible further decrease systematic error of 0.15% by measurements of standard reference materials and introducing corrections of K_ε coefficients, but it would require perfectly machined specimens whose permittivity is defined with precision better then 0.15%.

Dielectric loss tangent uncertainty depends on many factors, mainly on of Q-factor measurement uncertainty and the value of the electric energy filling factor. For properly chosen sample thickness it is possible to resolve dielectric loss tangents to approximately 2×10^{-5} for Q-factor measurements with an accuracy of about 1%. Theoretical uncertainties of dielectric loss tangent measurements for our 3.37 GHz resonator are shown in Fig. 2. As one can expect these uncertainties increase with decreasing values of dielectric loss and permittivity.

Table 2
Complex permittivity measurements of single crystal standard reference materials using split post dielectric resonators

	SPDR data		Reference data[8]			
f (GHz)	ε'_r ±0.3%	tanδ ±2E-05	ε'_{rr} ±0.1%	tanδ ±5%	$\frac{\varepsilon'_{rr}-\varepsilon'_r}{\varepsilon'_{rr}}$(%)	Material
3.9	9.420	2.40E-05	9.400	1.0E-05	0.21	Sapphire
1.4	4.448	1.15E-05	4.443	1.5E-05	0.11	Quartz
2.0	4.454	1.82E-05	4.443	1.5E-05	0.25	Quartz
3.9	4.443	2.58E-05	4.443	1.5E-05	0	Quartz
5.5	4.439	3.40E-05	4.443	1.5E-05	0.09	Quartz

Table 3
Results of measurements of stacked polymer films

h (mm)	ε'_r	tanδ	Number of stacked films
0.100	3.19	49E-04	1
0.201	3.20	50E-04	2
0.303	3.20	50E-04	3
0.406	3.20	49E-04	4
0.511	3.19	49E-04	5
0.616	3.18	50E-04	6

3. Experiments

We choose single crystals as standard reference materials since they have precisely determined permittivity and very low dielectric losses. The first property enables to assess experimentally measurement uncertainty of permittivity in SPDR's and the second enables to assess loss tangent resolution.

In Table 2 there are results of complex permittivity measurements of sapphire and quartz samples. Results are compared to the reference values.[8–13] It is seen that all permittivities agree within specified measurement uncertainties. Also loss tangent values agree within specified loss tangent resolution. For very low loss materials, like sapphire or quartz, Q-factor of the SPDR with a sample were often greater than the Q-factor for the empty SPDR. In spite of this evaluated dielectric loss tangent values were greater than zero due to proper Q_c and Q_{DR} corrections. It should be mentioned that loss tangent resolution for the thickness of a sample close to the height of air gap of SPDR's is the order of $\pm 2E$-05 and it is predominantly limited by Q-factor uncertainties, and Q-factor values of empty SPDR's. As loss tangent of the sample increases, accuracy of loss tangent also increases converging to the Q-factor measurements accuracy for medium loss samples.

For materials having larger dielectric losses measurement results of the dielectric loss tangent are usually very consistent. That can be seen in Table 3 where permittivity and loss tangent values are shown for stacked polymer films. One can observe that measurement results are independent of the number of films, because SPDR method is not sensitive to the presence of air gaps between stacked films.

4. Conclusions

The split-post dielectric resonator has been shown to make a useful, accurate and convenient contribution to dielectric metrology. It offers accurate measurements with quantifiable uncertainties for wide ranges of permittivity and loss in frequency range 1–10 GHz that plugs a gap in the frequency coverage of existing methods. The method is especially useful for measurements of flat laminar specimens without any need for machining of their shape.

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